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GHOSTS AND OCULARS.

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The visibility of celestial objects, as of objects in general, depends on brightness contrast with the background. The more general light in the field of vision the more must come from a particular object to bring it within range of the eye. The average total brightness of the sky has been shown by Yntema (Gron. Pub.# 22) and van Rhijn (Ap. J. **50**, 374) to be 0.13 or 0.14 of a 1st magnitude star per square degree. Considering the naked eye as an optical instrument of about 5 mm. aperture, one finds equality of the image with this background at about $6^{m}.5$, which suggests that the contrast required for visibility is fully $0^{m}.5$. With telescopes the case is less favorable since 35 to 40 per cent. of the light is generally lost in objective and ocular, substantially half in the latter. The loss by reflection goes chiefly outside the instrument as regards the objective. The loss in the ocular is chiefly scattered on diaphragms and other surfaces close to the eye thus lighting up the field, and part actually passes directly to the eye. The result is a material loss of visibility in faint objects due to the enhanced brightness of the field, and sometimes the appearance of false images.

This investigation is directed at the magnitude and distribution of the reflected components of the light and particularly those which are within the ocular, too often constructed without regard to the effect of its reflections on the field viewed by it.

For the present purpose a ghost in an optical instrument may be defined as an image, real or virtual, formed by the reflection of light from the surfaces of the system. It is commonly far out of focus so that it merely forms a hazy spot in the field, or diverging still more widely merely serves to scatter a small amount of light over the whole field. Under certain conditions it may, so to speak, materialize as a fairly bright image, in which case it becomes a nuisance. Most visible ghosts are referable to the ocular of the instrument, whence the desirability of considering the ocular, which has been grossly neglected.

Any beam of light incident normally on a lens system suffers reflection at each refracting surface according to the perfectly well known law established by Fresnel. In any lens system of free surfaces $I' = I\,(1 - K)^n$ where I' is the transmitted light and k is Fresnel's factor $\left(\frac{\mu - \mu'}{\mu + \mu'}\right)^2$. The transmitted light is then, for unity incident light, $(1 - k)^n$. For practical purposes k varies from a scant 4% in light crown glass to 6% or a little more in dense flint, averaging perhaps 5% in lenses as they are found in practice. The total light thus reflected, and lost to the incident ray as such, evidently increases rapidly with the number of surfaces, and runs about as follows:—

TABLE

Kind of lens	Number surfaces.	Tr.	Ref.	No. 2nd order Ghosts
Single lens	2	.90	.10	1
Double (ordinary objective)	4	.81	.19	6
Triple objective (Cooke, Heliar, etc.)	6	.73	.27	15
Uncemented pair achromats (Ross, Celor, etc.)	8	.65	.35	28
5 lens objective (Beck Isostigmar)	10	.59	.41	45

Measured transmissions are often several per cent. below these figures.

This last case includes prism glasses and ordinary terrestrial telescopes. No account of loss from absorption as such is here included. Absorption ordinarily amounts to between 1 and 2% for 1 cm. thickness following the usual exponential law for greater thicknesses. It does not bear any particular relation to the index of refraction until one comes to very dense flints when the absorption is specific in the blue and violet. More light is lost in a single reflection than in several cm. of glass. For our present purpose no account is taken of loss from imperfect polish, and dust, which may produce some scattering and obstruction of light.

Now the ghosts, visible as such, or spread over the whole field, have an aggregate intensity equal to the total light lost by reflection, and they may be divided into two series, odd and even with respect to the number of reflections. All the odd series are directed toward the incident ray and in themselves produce no visible effect, save as they may be reflected from the inner side of diaphragms. In some instances they may become objectionable, since the diaphragm is likely to be as good or better a reflector than a glass surface. The coefficient of reflection generally is nearer 10 than 5 % for it is a very dense and perfect black indeed that falls below 5%. The ghosts of even order, however, furnish light traveling with the incident ray and consequently the even order ghosts bring light directly into the field and may become distinctly visible.

In general the total of the reflected light is divided into ghosts of the odd and even series in the approximate ratio $k : k^2$. If P be the total reflected light, then any 1st order ghost must be less than $\frac{P}{n}$, any 2nd order less than $\frac{2kP}{n(n-1)}$.

More precisely the ghosts of the first order odd series have intensities kI, $kI(1-k)^3$, $kI(1-k)^5$ and their sum is, putting $t = (1-k)$

$$\Sigma i = kI\,(1 + t^3 + t^5 - - - + t^{2n-1})$$

while the second order ghosts, even series, have an approximate total intensity $k^2\,It^n$, since each ray concerned has to pass at least n surfaces, in average somewhat more.

INTENSITIES OF INDIVIDUAL GHOSTS.

Since ghosts of the 3rd order have coefficients in k^3 and pass forward they may practically be neglected, and those of the 4th order likewise, having, even if in focus, magnitudes at least 14–15 m. less than the primary image from which they are derived.

Of the n ghosts of the 1st order none can cause trouble save indirectly, as already shown.

There are $\frac{n(n-1)}{2}$ 2nd order ghosts and the maximum brightness of any one ghost in focus must be at least 7.5 m. below its primary. In ordinary objectives and photographic doublets the 2nd order ghosts are 7.5 to 8.0 mg. below the primary even when in focus. The difference can readily be measured by bringing the principal ghost to the area of its primary by changing focus and then using a photographic wedge to measure the relative intensities. The difference in brilliancy comes out at approximately 7–8 m.

Cemented surfaces may practically be left out of account since for ordinary pairs $\left(\frac{\mu - \mu'}{\mu + \mu'}\right)^2$ does not exceed 0.0009 and in a ghost of the 2nd order this small quantity must be multiplied kt^{n-2}, since there is one free reflection and 2 surfaces are abolished by cementing. The available light in a ghost from a cemented doublet is then

$$.0009 \times .05 \times .90 = .00004$$

i.e., such a ghost is 11 mg. fainter than its primary, and is barely discernible even under laboratory conditions.

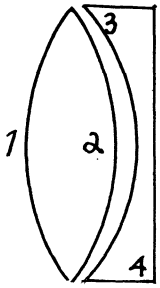

FIG. 1.

Taking up now the typical case of a telescope objective with reference to ghosts, we may, in Figure 1, sketch the form of common shape, numbering its surfaces 1, 2, 3, 4. It will give 6 second order reflections from the following combinations of surfaces

2 – 1
3 – 1
4 – 1
3 – 2
4 – 2
4 – 3

Of these the first four commonly give real images somewhere along the axis, the last two correspond to virtual images in the direction of the incident light, the light being spread to the rear as is usual where a diverging system does the reflecting. The first 3 reflections obviously come to focus relatively near the objective in lenses of ordinary form, and thus merely scatter light of which some actually enters the visible field. The remaining reflection 3–2 is between surfaces optically almost parallel so that after its second reflection the ray is nearly parallel to the transmitted ray incident on 3, and comes to about the same focus.

It is the ghost thus produced which is occasionally troublesome in ordinary objectives, and frequently so in photographic doublets of the Petzval construction. Under any conditions where two consecutive surfaces are of substantially equal curvature the ghost should be looked out for, whether the surfaces are those of an air space or are formed by a lens element of very slight refraction. The exact position of this ghost from the intermediate surfaces of an objective may be quickly found graphically. From any point of surface 2 draw a line to the focus of the front lens considered by itself. The intersection of this line with surface 3 at once gives the angle of incidence there from which the reflected ray can be traced backward and then through the rear lens to its new focal point. If one traces a ray in fact through any combination an inspection of the angles of incidence at the various surfaces will generally show at once where any possible danger from ghosts lies. In dealing with a completed lens a very brief inspection with a bright light in the field or the exposure of a photographic plate will tell the story. In a simple doublet all 6 reflections can be photographed and in a triple or quadruple combination most of the ghosts will show although some may be lost by overlapping.

Effect of the Ghosts.

In an ordinary objective the result of the 2nd order reflections is slightly to brighten the field, except that the air space ghost may appear as a spot. In the photographic case the effects are more conspicuous and where, as in uncemented 4-lens combinations, the total intensity of the reflections may amount to something over 2% of the total light, the clarity of the image may be materially impaired and in photographing nebulae false images may appear. Such systems give pictures distinctly less crisp and brilliant than does the single cemented landscape lens. In the visual case a visible ghostly image requires a surface brightness greater than the background of the sky, in other words comparable with the star disc of the *minimum visibile* in the instrument used.

As the 2nd order ghost in focus is about 7–8 m. fainter than the primary it must be thrown out of focus enough to dim it below practical visibility. In the objective the angle a between the ray incident on surface 3 and its return after two reflections is

$$a = 2\,(i - i')$$

where i' is the angle of reflected incidence on surface 2. Calling this Δi and noting that the deviation of a ray is for small refractions constant, the variation of focal length ΔF is approximately

$$\Delta F = F\frac{\sin \Delta i}{\sin i}$$

The situation is just as if the original ray incident on 3 were tilted through an angle Δi swinging the line to focus along with it.

It should be noted that where $(i - i')$ is large, as for large values of $\varrho_2\ \varrho_3$, ΔF increases, so that for a given focal length high absolute curvature, as in some flint-ahead combinations and objectives of Fraunhofer form, gives larger variation of focus, while in Gaussian objectives where $\varrho_2\ \varrho_3$ are both large and widely different, the ghosts are widespread and the field exceptionally dark, as in the Princeton 9″.6 glass, designed by Professor Young.

In actual practice, objectives of ordinary glasses computed to meet the sine condition give ϱ_2 and ϱ_3 differing by 2–3% which is more than enough to wipe out the visible ghost.

Since, *caeteris paribus*, $(i - i')$ depends on the ratio between ϱ_2 and ϱ_3, roughly,

$$\Delta F = F\frac{2\,\Delta\varrho}{\varrho_3}$$

Whence a difference of 1% between ϱ_2 and ϱ_3 will shift the focus about 2% back or forward according as $\varrho_2 < \varrho_3$ or the reverse. Thus is a telescope of 100 cm. focus and 10 cm. aperture a ghost image of 1″ radius has a diameter of 0.01 mm. The intersect of its cone of light at 2 cm. from focus would be 2 mm. and the surface intensity would be $\frac{1}{40000}$, i.e. 11 mg. below the original, or at least 18 mg. below the primary, a negligible figure even allowing for the greater visibility of an enlarged area.

It is in oculars that troublesome ghosts are most frequently met, sometimes distinctly visible, often brightening the field.

Consider a simple field-lens, A, Figure 2, of radii r, r' receiving substantially parallel light which is reflected from its rear surface. It is subjected to 2 refractions and one reflection and comes to focus as if the lens were replaced by a concave mirror of focus f defined by

$$\frac{1}{f} = \left(\frac{\mu}{r'} - \frac{\mu - 1}{r}\right)$$

For $r = \infty$, the common form of field lens, and common crown glass, $f = \frac{1}{6}F$ approximately, F being the ∞ focus of the lens. This defines the position of the 1st order ghost. The 2nd order ghost due to 2 reflections and one refraction lies at a closely similar distance, f' to the rear. The loci can be readily found by holding a large lens at a fair distance from a lamp and looking for the ghosts with a slip of ground glass. The position of these 2nd order ghosts is not greatly changed by distributing the total curvature over both sides of the lens, while with thick lenses the ghosts lie closer in toward the vertices, and in achromats further out.

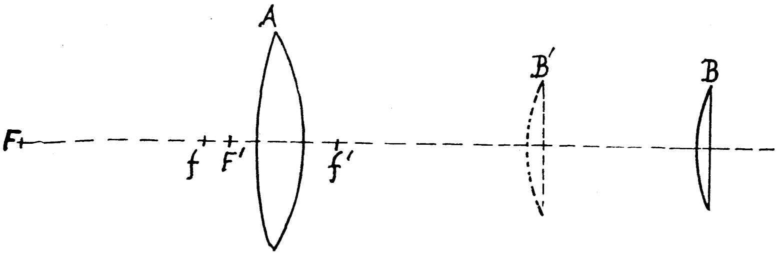

FIG. 2.

It is this 2nd order field-lens ghost that is liable to cause the most trouble. In a 2-lens ocular some 10% of the whole incident light is scattered behind the stop, which may be a large quantity compared with the object under scrutiny, and seriously affect the dark adaptation of the eye. A variable stop, could often be used to advantage.

The ghost focussed at f' lies inside the focus F' of the ocular and is visible as a somewhat blurred spot the more conspicuous the nearer to focus. In a Ramsden ocular of the theoretical form $f = f' = F$ the ghost is very obtrusive.

For example, in the rather thick field lens of a big positive ocular I found the rear ghost at less than $\frac{1}{10}$ the focal length back of the rear vertex. And a very little experimenting with a pocket lens will enable one to realize that such a ghost is not out of focus enough to count. As the ocular is shortened by moving the eye lens B to the conventional separation of $\frac{2}{3}$ f the ghost goes considerably out of focus and appears as a large hazy spot increasing as B moves toward B'. An ocular with sliding fit between field and eye lens gives a very instructive view of the way the ghost comes to approximate focus as the lenses are separated, especially if one half-silvers a surface of the field lens, when the effects are very brilliant.

The best place to study ghosts is in an ordinary prism glass where a prism surface lies just ahead of the flat face of the field lens with three other optically parallel faces in reserve. Three ghosts are conspicuously bright. One, the ordinary field-lens ghost, one from the reflection between the front of field lens and prism face, and the third by reflection from back of field lens and prism face. Besides these at least two others rather faint and dodging quickly about in the field can be made out, chargeable to the adjacent rear prism faces.

With respect to oculars themselves the general rule applies regarding the number and effect of reflecting surfaces, with the same proviso that the 2nd order ghosts are the ones seen, while the 1st order may cause troublesome reflections from the mount.

The fact is that oculars have received very scant treatment at the hands of practical opticians despite the fact that half the light lost in the telescope is generally lost in the ocular, and that it is the seat of no inconsiderable trouble from stray light. It is not the loss of light in the image that hurts, but the effect of the light lost. What is the use of an objective of the highest corrections producing a perfect primary image if that image is magnified by an eye piece of indifferent definition and appears shrouded in a haze of scattered light?

The ordinary ocular has a strongly curved field, somewhat distorted and imperfectly corrected for color. Despite the fact that very material improvements have been made, mostly to meet the severe requirements of microscopic vision, telescope makers stick quite closely to the old Huyghenian and Ramsden forms, chiefly perhaps because these are composed of simple plano-convex lenses and are therefore easy and cheap to make. There is a tendency, too, toward struggling for an abnormally wide field at the expense of definition. Nearly all eye pieces, and particularly these mentioned, have a notably curved field or else are viciously astigmatic in the outer zone, and these faults increase with the angle of view. The eye can take in above 40° but not much more than half this angle is anywhere nearly simultaneously sharp and anything materially over 40° is only attained by peering around the edges of the field.

Fortunately the only cases in which wide field is really useful are those in which the magnifying power is relatively low, which enables the accommodation of the eye to help out. The curvature of the image derived from Petzval's equation is

$$\frac{1}{R_o} - \frac{1}{R_i} = \frac{1}{\mu_1 f_1} + \frac{1}{\mu_2 f_2} + \text{etc.} = \frac{1}{\Sigma \mu f}\,;\ \text{when } R_o \text{ is large, } \frac{1}{R_i} = \Sigma \frac{1}{\mu f}.$$

Where R_o is the radius of curvature of object, R_i of image, μ, as usual, the index of refraction, and f_1, f_2, etc. the focal lengths of the equivalent thin lenses obtained in the ordinary way. It hence is obvious that the radius of curvature of the sharp image in a single crown glass lens is practically $\frac{3}{2}f$ while in a 2-lens combination widely separated, as in the common forms, it is about $\frac{3}{4}f$. Other oculars lie between these limits. Evidently highly refracting glasses relieve the situation a little, but they are rarely used, being rather harder to get than common crown. Since the easy range of accommodation of the eye except in early youth is not much over 2 diopters the extent to which it can compensate the curvature of the image is rather unsatisfactory, except in low power eyepieces. Conrady (M. N. *78*, 445) in discussing this subject has shown that for a total field of 40° the sharp field fails in ordinary eyepieces for focal lengths under an inch, while a carefully designed achromatic combination, e.g., # 4, 5, 9, Figure 3, will reduce this figure to about $\frac{1}{2}$ in., and he further shows a very interesting possibility in the construction of anastigmatic eyepieces to give at once a field at once flat and free from marginal astigmatism. It is to be hoped that this masterly optical investigator will carry out his own suggestions, but even without anastigmats it is quite feasible to get greatly improved eyepieces of moderately large field and with interference from reflections greatly reduced.

The freest of all from reflections are, of course, the single lenses, of which the more common forms are shown in numbers 1 to 5 of Figure 3. The simplest of them is the ordinary plano-convex lens such as is obtained by removing the field lens of a Huyghenian ocular. It gives somewhat increased light but a very small sharp aperture, not over 10°. A little better field and similar illumination is given by the solid eyepiece commonly known under the name of Coddington, although due to Sir David Brewster. It is derived from a glass sphere by removing a thick equatorial belt and then cutting an equatorial groove for a stop, down to a diameter of something less than half the radius of the sphere. Such an ocular of crown glass is of focus about three halves the radius of the sphere and the field is rather larger than that of a simple lens, but badly chromatic toward the edges.

The next step in improvement of results from the single lens is the simple achromat, # 3, such as is made for an eyepiece by Cooke and others. When well designed and preferably of high index glasses it gives a field of from 15° to as high as 20°, free of color, but not fully orthoscopic. Next, one comes to the triple achromats, of which # 4 is a typical example, as made by Zeiss, Steinheil, and others. The field of such a triplet is usually good for 20° to 30°, colorless, quite flat

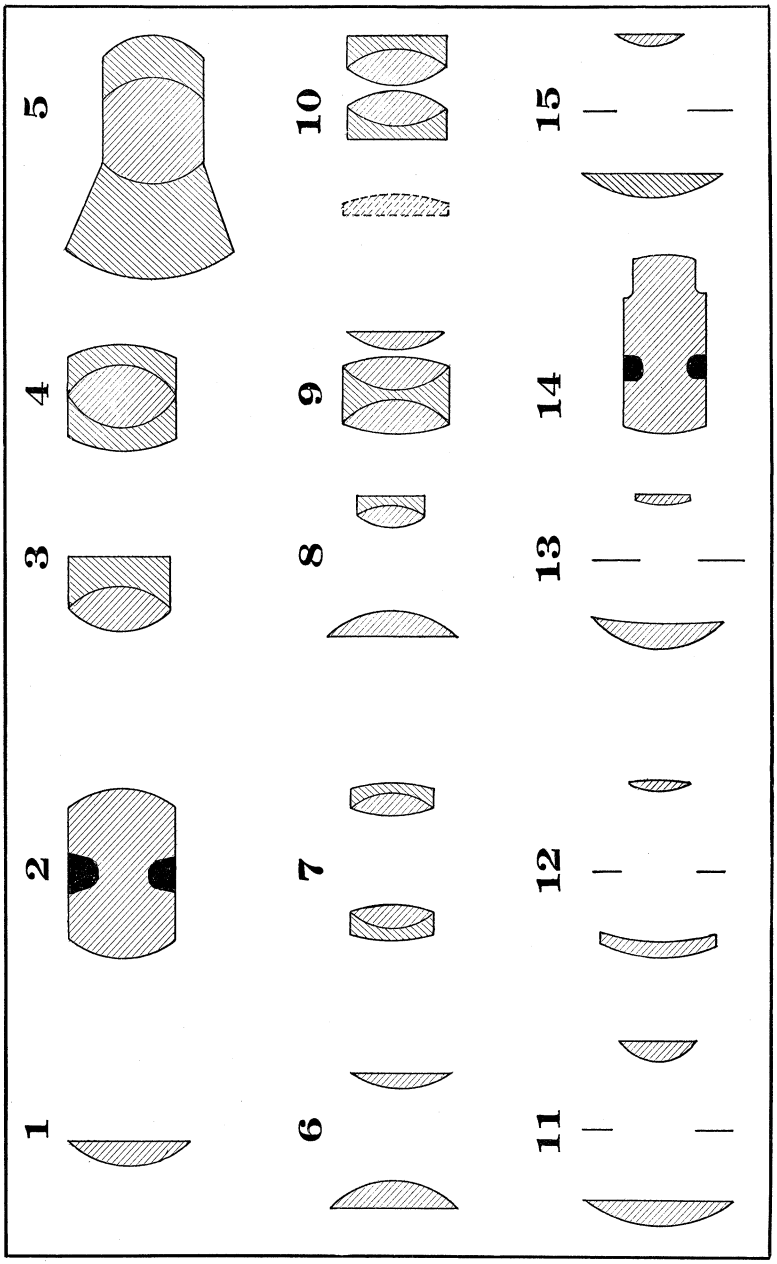

Fig. 3.

and orthoscopic. It is a form which when properly designed makes one of the most useful of eyepieces, beautiful in definition and remarkably free of ghosts.

A modification, # 5, is the so-called monocentric eyepiece of Steinheil, in which advantage is taken of thickness of one of the components to secure better flatness of field and also orthoscopy. Thickness gives an additional disposable factor for this purpose in $\frac{d\,i}{d\,h}$ with respect to the last surface. This is a device practically very useful, and employed both in other triplets and in photographic lenses. The monocentric eyepiece has a field of about 32°, remarkably flat and orthoscopic, and like other forms of triplet has a fairly good eye distance. All these oculars are practically free of ghosts.

The positive doublets are commonly troublesome as respects ghosts. As already mentioned, the Ramsden, # 6, even at its common separation gives a large hazy ghost as well as some residual color. It has an ordinary field of about 35°. Its achromatic form, # 7, has a slightly better aperture, is more nearly orthoscopic and is quite free of color. The ghost is much less troublesome, for the reason already mentioned, that it lies further out of focus.

The ordinary Kellner eyepiece, # 8, is the worst of those in common use with respect to ghosts, although otherwise valuable, since the field runs to as high as 45° and the orthoscopy and color are both excellent. The trouble comes from the field lens being so nearly in the focus of the eye lens. Sometimes, indeed, the eye lens focus lies within rather than outside the plane surface of the field lens. Just what the original form of Kellner was is somewhat dubious, but probably it had a double convex field lens which would not, however, improve it in the matter of ghosts.

Two other forms of positive doubtlets are worth mentioning, as they are not only unusually free of ghosts but possess singularly valuable properties. One of them is the so-called orthoscopic ocular of Zeiss and Steinheil # 9, consisting of a thick triple achromat with a plano-convex eye lens almost touching it. The field is about 40°, is quite free of visible ghosts for obvious reasons, and is quite flat and orthoscopic, with exquisite definition and a fairly good eye distance. The other form, # 10, is a type of long relief ocular much used in artillery sights. It consists of two simple achromats with practically equi-convex crowns placed close together with the crowns almost touching. It is used either with or without a field lens of relatively long focus. When so mounted the field is over 40°, beautifully flat and orthoscopic with the finest of definition clear up to the edge. The focus of the field lens is so long that no ghost is noticeable, while without the field lens the slightly reduced field is still of high quality. The eye lenses being large allow a relief practically equal to the focal length, and sometimes more. It is a combination of which good astronomical use might be made in finders. These two oculars, #9 and #10, are so much superior optically to the ordinary forms that it is a pity they are not more used.

Finally we come to the group of so called negative oculars, all substantially ghost free, and modifications of the Huyghenian type. The freedom from visible ghosts results from the relatively considerable separation of the field and eye lens whereby the 2nd order ghost from the former falls much beyond the focus of the latter and merely scatters a little light over the field. No. 11 is the ordinary Huyghenian with ratio between the focal lengths of field and eye lens of 3:1 to 2:1, giving fields generally of 40° to 45° a little distorted toward the edges, with pretty efficient pseudo-achromatism and generally good definition. No. 12 is the Airy form with the lenses in the ratio of 3:1 as before, but with the field lens a rather strongly curved meniscus, and the eye lens crossed. There is a distinct gain in marginal field and a generally slightly better performance at wide angles.

No. 13 is the Mittenzwey eyepiece, manufactured by Steinheil and others. It is quite similar to the Airy except that it is often made on the 2:1 ratio, the front meniscus is less curved, and the eye lens plano-convex. I can say from experience that these eyepieces are most admirable. The definition is good and they give a preposterously big field, near 50°. This is perhaps the most practical form of eyepiece where the maximum possible field is required, the optical properties being excellent and the meniscus curves being within the bounds of easy construction.

No. 14 is the solid Huyghenian eyepiece of the late R. B. Tolles which he frequently made both for telescopes and microscopes. It is practically a Huyghenian eyepiece with curvature ratios of $1\frac{1}{2}$: 1 between field and eye lens, the eye end being of course convex. A groove stop, in diameter about $\frac{1}{2}$ the focal length, is cut around the long lens at about $\frac{1}{3}$ its length from the field end, practically in the focus of the eye end, of which the radius is substantially half the focal length. This ocular seems to have passed out of use with the death of its original constructor, but it possesses remarkable qualities in that it is quite free of reflections, gives a wide and beautifully sharp field and the brilliancy which accompanies the single lens. Its field is nearly as big as that of the Mittenzwey eyepiece. I recently tried out a $\frac{1}{4}''$ ocular of this kind beside two Mittenzwey oculars and found it distinctly superior to them in brilliancy and at least equal in definition of any eyepiece I have ever seen. It seems to be really a most admirable form, particularly for small telescopes where it is necessary to save all the light possible without sacrifice of other qualities.

Finally I note in #15 a modification of an ordinary Huyghenian ocular which I have recently tried with reference to improving the color correction. It is well known of course that the objectives of telescopes are usually over-corrected for color in order to compensate for the under-achromatism of the eye combined with its eyepiece. This over-achromatization can, of course, compensate accurately only one particular degree of under-correction, that which corresponds to some selected power, or more precisely, to some selected aperture of the eye, the achromatic error diminishing with the diameter of the emergent pencil from the ocular. The point chosen is usually that which corresponds to about .5 mm. diameter in the emergent pencil. For all lower powers the objective is under-corrected, for higher powers over-corrected.

Therefore it will be of distinct advantage to have the lower oculars somewhat over-corrected and the higher ones under-corrected. Now it is easy to apply under-correction to a telescope ocular in practically the same way that it is applied to the compensating ocular used with apochromatic microscope objectives. Several years ago I had applied the scheme in correcting the residual color in gun sights, and recently tried out the same principle by using, in a Huyghenian of 2:1 ratio, a field lens of highly dispersing flint. The obvious result is to pull down the focus for the blue end of the spectrum, initially too long on account of over-correction.

I have tested this particular ocular of 5 mm. equivalent focus on a much over-corrected $3\frac{1}{2}''$ telescope and a $3''$ of about the usual degree of over-correction with the expected result of cutting down the blue outstanding light, and reducing the in-focus image of an artificial star to the normal hue of the secondary spectrum. The plan gives every appearance of working well, at least for objectives of moderate focal length, and it would seem possible by following it out very materially to improve the color corrections otherwise attainable. Since one cannot achromatize for all powers the best plan would seem to be to achromatize for the most generally useful power, and, since the oculars are far the least expensive part of the telescope outfit, to improve the color correction through modification of these, under-correcting the high powers and over-correcting the low ones. Inasmuch as the eye loses in acuity very rapidly at apertures below 1 mm. it would seem advisable to correct objectives for a rather lower power than is now customary.